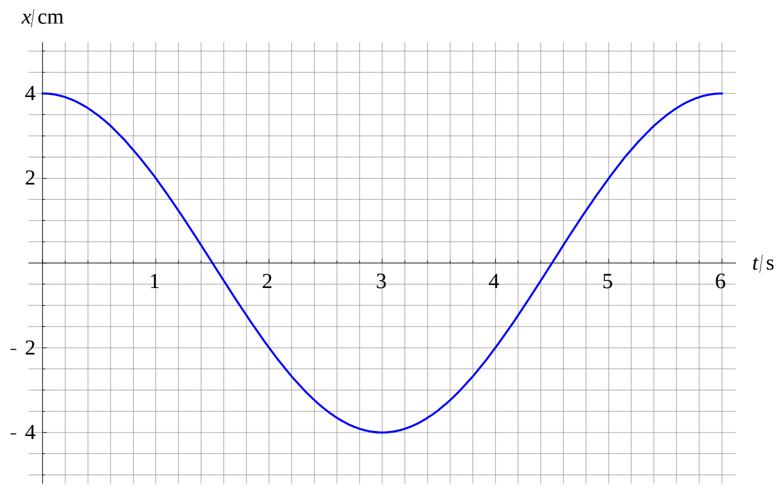
Problem of the week

Simple harmonic oscillations (HL only)

(a) The graph shows the variation with time of the displacement of a particle from its equilibrium position.



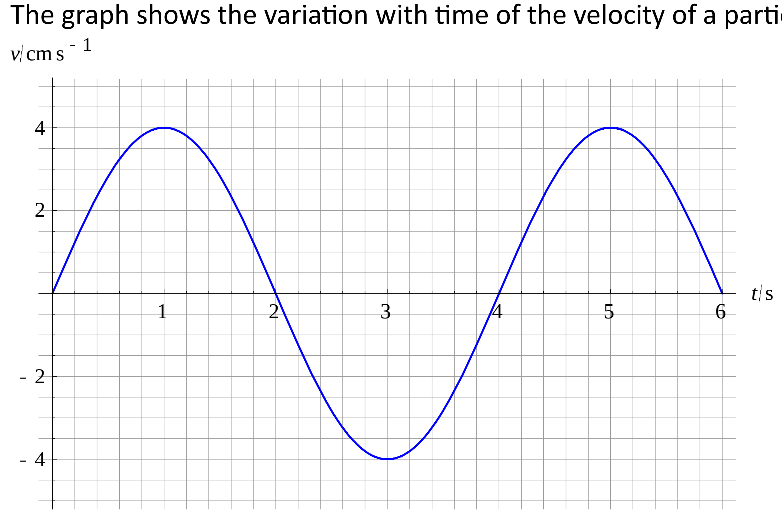
(i) Write down the equation for the displacement in the form $x = x_0 \sin(\omega t + \varphi)$ where x_0, ω and φ are to be determined.

(ii) Calculate the maximum speed of the particle.

(iii) Determine the speed of the particle when the displacement is 2.0 cm.

(iv) Verify algebraically that the first time when the displacement becomes 2.0 cm is about 1 s.

(b) The graph shows the variation with time of the velocity of a particle performing SHM.



(i) Estimate the amplitude of oscillations.

(ii) Estimate the area under the curve from $t = 0$ to $t = 2$ s.

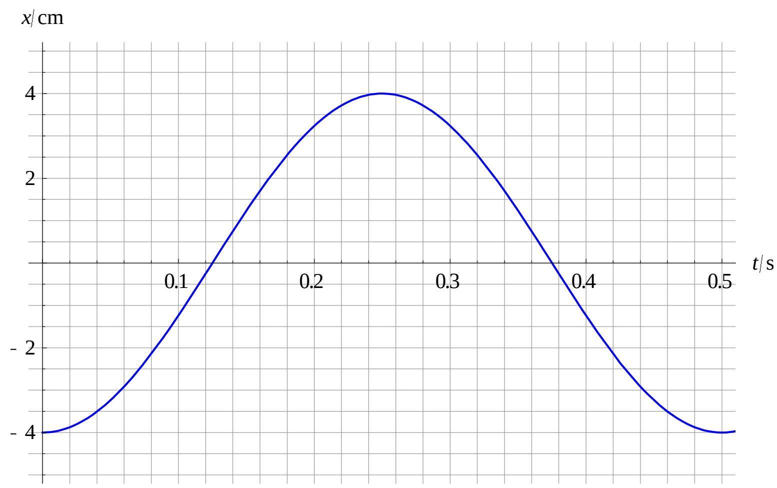
(iii) Determine the displacement when the velocity is 3.0 cm s^{-1}.

(c) In a SHM the maximum acceleration is 24 m s^{-2} and the maximum speed is 2.0 m s^{-1}. Determine

(i) the period of the oscillations,

(ii) the amplitude of oscillations.

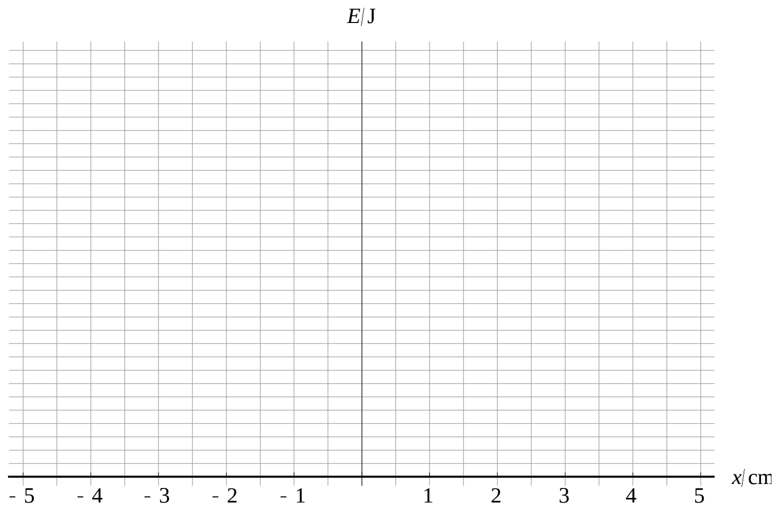
(d) A block of mass 1.2 kg is attached to a horizontal spring. The graph shows the variation with time of the displacement of the block.



(i) Determine the spring constant.

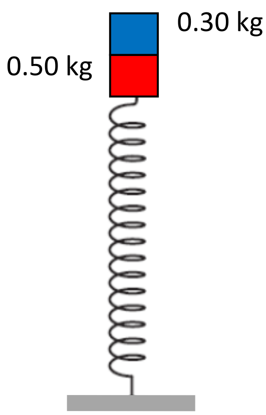
(ii) Draw, on the same axes, graphs to show the variation with time of the kinetic energy of the block and of the elastic potential energy of the spring.

(iii) Draw, on the same axes, graphs to show the variation with displacement of the kinetic energy of the block and of the elastic potential energy of the spring.



(iv) The mass of the block is quadrupled. Suggest how the graphs in (ii) and (iii) change, if at all.

(e) Two blocks of mass M = 0.50 kg and m = 0.30 kg are in equilibrium on top of each other and on top of a spring of spring constant k = 120 N m^{-1}. The red block is firmly attached to the spring. The blocks are pushed down a distance d and released.



(i) Calculate the compression of the spring at equilibrium.

(ii) Determine the largest value of d such that the blocks do not separate.

(iii) Suggest how the answer to (ii) would change, if at all, if the red block was on top.

Answers

(a)

(i) The amplitude of oscillations is 4.0 cm so $x_0 = 4.0$ cm. The period is 6.0 s so $\omega = \frac{2\pi}{T} = \frac{2\pi}{6.0} = \frac{\pi}{3}\ \text{s}^{-1}$. At $t = 0$, $x = 4.0\text{cm}$ so $4.0 = 4.0\sin(0+\phi) \Rightarrow \sin\phi = 1 \Rightarrow \phi = \frac{\pi}{2}$.

Hence, $x = 4.0\sin(\frac{\pi t}{3} + \frac{\pi}{2})$.

(ii) $v_{max} = \omega x_0 = \frac{\pi}{3} \times 4.0 = 4.19 \approx 4.2\text{m s}^{-1}$.

(iii) $v = \omega\sqrt{x_0^2 - x^2} = \frac{\pi}{3} \times \sqrt{4.0^2 - 2.0^2} = 3.63 \approx 3.6\text{m s}^{-1}$.

(iv) We must solve the equation $2.0 = 4.0\sin(\frac{\pi t}{3} + \frac{\pi}{2})$. This is a trigonometric equation; it has many (infinite) solutions so we must be careful. We must find the least positive solution. One solution is: $2.0 = 4.0\sin(\frac{\pi t}{3} + \frac{\pi}{2}) \Rightarrow \sin(\frac{\pi t}{3} + \frac{\pi}{2}) = \frac{1}{2} \Rightarrow \frac{\pi t}{3} + \frac{\pi}{2} = \frac{\pi}{6} \Rightarrow t = -1.0\text{ s}$. This is not what we want. The next solution is: $\sin(\frac{\pi t}{3} + \frac{\pi}{2}) = \frac{1}{2} \Rightarrow \frac{\pi t}{3} + \frac{\pi}{2} = \frac{5\pi}{6} \Rightarrow t = +1.0\text{ s}$ which is what we want.

[It is unlikely that a question of this kind will appear on IB exams, but it is good to have some practice with this because it reinforces understanding and makes nice connections with what you learn in math classes. After all, what is the point of learning all this in math classes if you cannot apply it anywhere!]

(b)

(i) The period is 4.0 s so $\omega = \frac{2\pi}{T} = \frac{2\pi}{4.0} = \frac{\pi}{2}\ \text{s}^{-1}$. $v_{max} = 4.0\text{ cm s}^{-1}$ and $v_{max} = \omega x_0$, hence $8.0 = 4\pi \times x_0 \Rightarrow x_0 = 2.546 \approx 2.6\text{ cm}$.

(ii) The area is twice the amplitude and so equal to 5.1 cm.

(iii) From $v = \omega\sqrt{x_0^2 - x^2}$ we find $3.0 = \frac{\pi}{2}\sqrt{2.546^2 - x^2}$ and so $x = 1.54 \approx 1.5\text{ cm}$.

(c)

(i) $v_{max} = \omega x_0$ and $a_{max} = \omega^2 x_0$ so that $\frac{a_{max}}{v_{max}} = \omega = \frac{24}{2.0} = 12\ \text{s}^{-1}$ and so

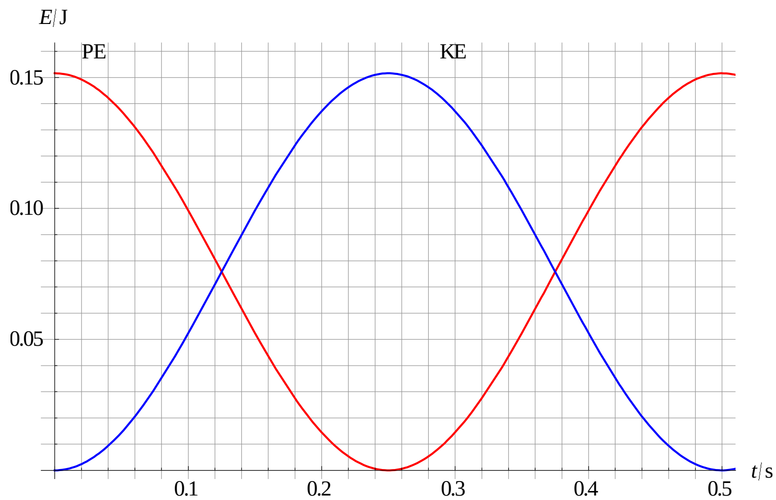
$T = \frac{2\pi}{\omega} = \frac{2\pi}{12} = 0.524 \approx 0.52\text{ s}$.

(ii) $v_{max} = \omega x_0 \Rightarrow 2.0 = 12\ x_0 \Rightarrow x_0 = 0.17\ \text{m}$.

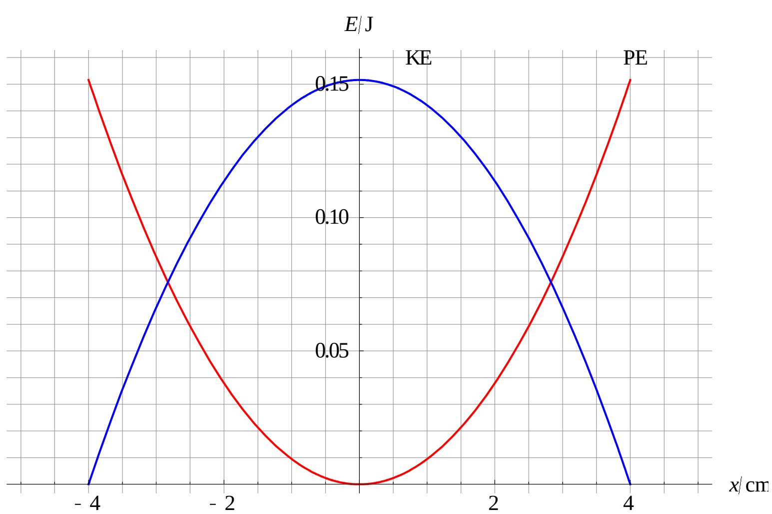
(d)

(i) $T = 2\pi\sqrt{\dfrac{m}{k}} \Rightarrow k = \dfrac{4\pi^2 m}{T^2} = \dfrac{4\pi^2 \times 1.2}{0.50^2} = 189.496 \approx 190\ \text{N m}^{-1}$.

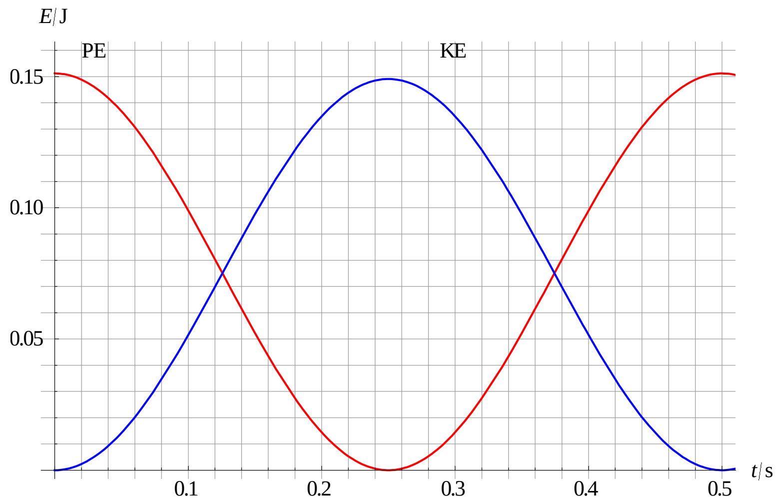
(ii)



(iii)



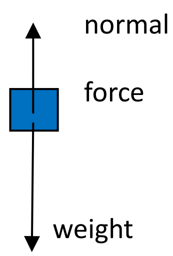
(iv) The elastic potential energy is given by $E_e = \dfrac{1}{2}kx^2$ and the kinetic energy by $E_K = \dfrac{1}{2}kx_0^2 - \dfrac{1}{2}kx^2$ so nothing changes in (iii). In (ii) we have $E_e = \dfrac{1}{2}kx^2 = \dfrac{1}{2}m\omega^2 x_0^2 \cos^2(\omega t)$ and $E_K = \dfrac{1}{2}m\omega^2 x_0^2 \sin^2(\omega t)$. The mass quadruples and so the angular frequency halves. Hence the maximum value remains the same but the period doubles. Hence:



(e)

(i) $(M+m)g = kx \Rightarrow x = \dfrac{0.80 \times 9.8}{120} = 6.5 \text{ cm}$.

(ii) When released the blocks will perform simple harmonic oscillations about the equilibrium position with $\omega^2 = \dfrac{k}{m+M} = \dfrac{120}{0.80} = 150 \text{ s}^{-2}$. The critical point is at the extreme top point of the oscillations. The forces on the top block are:



We have that $mg - N = ma$. The acceleration is given by $a = \omega^2 x = 150d$. The blocks will separate when $N \to 0$ i.e. when

$$mg = m150d \Rightarrow d = \frac{g}{150} = 6.5 \text{ cm}.$$

(iii) From the last equation, the mass cancels out so there would be no change.